Engineering Calculations Summary

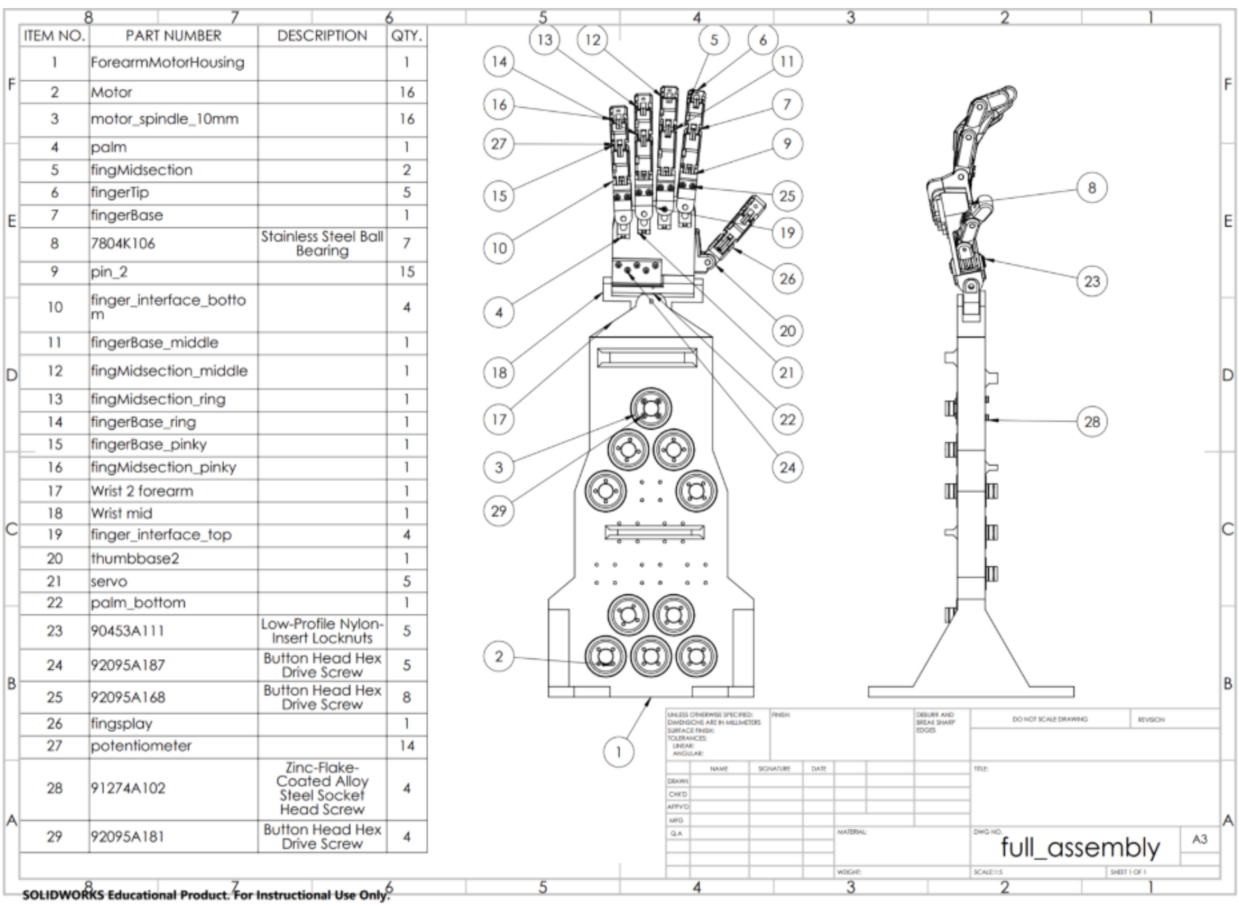
Humanoid Hand Team

Noah Enlow, Justin Alonzo, David Lutz, Joseph Maresh, Markus Steinebrunner, Tyler LeBeau

9/8/2025

Top Level Design Summary:

We have been tasked to create a humanoid hand that has as many degrees of freedom and is able to catch a ball and play the piano. Our solution is to create a tendon driven robotic hand that is about 1.5 times the size of a human hand so we can fit servos and tendon routing throughout the hand.



ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	ForearmMotorHousing		1
2	Motor		16
3	motor_spindle_10mm		16
4	palm		1
5	fingMidsection		2
6	fingerTip		5
7	fingerBase		1
8	7804K106	Stainless Steel Ball Bearing	7
9	pin_2		15
10	finger_interface_bottom		4
11	fingerBase_middle		1
12	fingMidsection_middle		1
13	fingMidsection_ring		1
14	fingerBase_ring		1
15	fingerBase_pinky		1
16	fingMidsection_pinky		1
17	Wrist 2 forearm		1
18	Wrist mid		1
19	finger_interface_top		4
20	thumbbase2		1
21	servo		5
22	palm_bottom		1
23	90453A111	Low-Profile Nylon-Insert Locknuts	5
24	92095A187	Button Head Hex Drive Screw	5
25	92095A168	Button Head Hex Drive Screw	8
26	fingsplay		1
27	potentiometer		14
28	91274A102	Zinc-Flake-Coated Alloy Steel Socket Head Screw	4
29	92095A181	Button Head Hex Drive Screw	4

Figure 1: Updated design

Our subsystems include: four fingers, a thumb, palm, wrist, and forearm. Our fingers (#5-7) and thumb (#20) oversee having the strength to press a piano key and also have a fast enough reaction time to catch a ball. The palm (#4) is built as a connection piece to connect the fingers with the wrist. The wrist (#17-18) has two different motions: forward and back and side to side. This will allow our hand to adjust its position when catching a ball. The forearm (#1) is going to be used as a base to connect the hand to a table as well as a place to keep all our motors. The forearm will hopefully also allow for the arm to be moved really easily throughout our client's lab.

System QFD

Project: Humaniod Hand
Date: 4/13/2025

1	Grip Force								
2	Time for actuation	-							
3	Approximate hand size	0	0						
4	Approximate hand weight	-	-	+					
5	Cost	-	-	0	-				
6	Degrees of freedom	0	0	+	0	-			
7	Easy to operate UI	+	0	0	0	+	-		
8	10,000 actuations	+	+	+	-	-	-	0	
9	Precision of motion	+	+	+	-	-	+	+	+

	Customer Needs	Customer Weights(1-5)	Grip Force between 120-160N	Time from full extension to full closure is 250-350ms	Approximate size of human hand	Apx 3-4kg	Cost of manufacturing<$1500	apx 20 DOF	Can be opperated by Lerner or Reza with a <10min demo	Each joint ensured up to 10k motions	Able to predict finger segment placement within +/- 5mm based on sensor data	1 Poor	2	3 Acceptable	4	5 Excellent
1	Strength	3	9	3	3	3	9	9	3	9	3		B		C	A
2	Speed	5	3	9	3	1	9	9	3	9	9			C	B	A
3	Accurate dimensions	2	3	3	9	9	3	9	3	9	3			A	C	B
4	Accurate weight	1	9	9	9	9	3	3	3	9	9		A	C		B
5	Budget	4	9	3	3	9	9	3	9	9	9	A		C		B
6	Many degrees of freedom	4	3	3	3	3	9	9	9	9	3	C			B	A
7	Uses stand form of power to function	5	1	1	9	9	9	1	9	3	3					ABC
8	Has basic and functional ui	4	1	1	1	3	3	3	9	1	9	C			B	A
9	Precision of motion	4	3	3	9	3	9	3	9	9	9			B		AC
	Technical Requirement Units		N	s	s	kg	$	deg	min	#	mm	Legend:				
	Technical Requirement Targets		##	0	##	4	##	20	10	10k	5	A		Shadow Hand		
	Absolute Technical Importance		##	##	##	##	##	##	##	##	##	B		Dex Hand		
	Relative Technical Importance		8	9	6	7	1	5	3	2	4	C		Optimus Hand		

Figure 2: QFD

For our customer requirements, we chose to focus on the following list of requirements given to us:

1. Biomimetic Dexterity – The robotic hand should be able to perform various complex manipulations, such as playing a song on the piano and catching a ball.
2. Human-like Size and Weight – To ensure realism and usability, the hand needs to mimic the size and weight of a human hand.

3. Adequate Strength – The hand must be capable of exerting a grip force slightly less than what the average person is capable of to effectively perform manipulation tasks.
4. Response Time – The actuation time from fully open to fully closed should closely resemble human reaction time enabling dynamic interactions such as catching a moving object.
5. Longevity – The design should support at least 10,000 actuation cycles per joint to ensure long-term operational use.
6. Ease of Use – The hand should be operable by researchers with minimal learning effort, featuring an intuitive user interface that requires no more than a 10-minute demonstration.
7. Cost Effectiveness – The total manufacturing cost should not exceed $1,500, despite having a combined project budget of $2,000, while maintaining high-quality materials.
8. Power Efficiency – The hand should function efficiently within standard electrical power limits (approximately 120V AC or 24V DC input).

Next our engineering requirements are shown in the following table:

Table 1: Enginering Requirements

Requirement	Target Value	Units	Tolerance	Justification
Grip Force	25-40	kg	±5	Matches human grip strength
Actuation Time	150-300	ms	±50	Ensures responsive movement
Hand Size	190x85	mm	±50x25	Comparable to human hand
Weight	2.5-3	kg	±0.5	Lightweight for usability
Degrees of Freedom	~20	#	±1	Maintains human-like dexterity
Actuation Cycles	10,000+	#	±250	Ensures durability
Cost of Manufacturing	<1,500	$	±250	Maintains budget constraints
Power Consumption	~120	V	0	Compatible with standard power
Precision and Accuracy	1	mm	±0.5	Maintains accurate motion control
User Interface Time	<10	min	±2	Easy setup and usability

Summary of Standards, Codes, and Regulations:

This device is intended as a university capstone for research purposes, and not for medical, industrial, or consumer deployment. There are no external standards or regulatory requirements that govern its design, fabrication, or operation. The team still followed good engineering practices throughout the design process, including proper documentation, safety awareness, and adherence to course requirements. If this project were to transition toward commercial or medical use, relevant standards would then include ISO/IEC standards for robotics and safety standards for electromechanical devices.

Summary of Equations and Solutions:

In this section we separated our sections by the topics our calculations followed last semester. In each section there is the person who did it and how these people got to our solutions. In the final section we have a summarized table of our calculations.

Power

Noah:

One of the critical considerations that needed to be addressed for the design of our robotic hand was power consumption. To address this, a python script was written which calculates the power consumption of the entire hand, as well as the power consumption of the individual components and subsystems. From this script and assuming standard servos used to actuate the hand, a maximum power draw of 107W was obtained. This corresponds to the hand gripping hard enough to completely stall the servos. 107W corresponds to running 1-2 desk lamps at the same time depending on the efficiency of the lamp. One of the benefits of this Python script is that it also serves as a record book of the electrical specifications of each component which the team can refer to whenever needed. This python script represents the electrical components of the hand as objects with individual attributes. For example, the motors inherit the "motor" class and take on their own unique values for voltage, current, and efficiency. After all of the electrical components have been defined, another class representing the hand as a whole takes all of those objects in as parameters, sums their individual power consumptions, and then returns the total power consumption of the hand. The script was written so that it would be easy to add new components or change the parameters of the components currently in there, making the script quite adaptable and useful even when new components are added/components are changed out for others.

Motors

Noah:

An important consideration to be made in the early stages of design is that of required motor torque. The hand will be operating via a tendon-driven system, with motors in the forearm controlling the finger movements. How much torque these motors will need to provide is a question that needs to be answered early in the design phase, as it will impact cost, weight, and size of the hand. As a result, a statics analysis of the hand was done to determine the required torque output of the motors. The assumptions of the analysis were that the hand would be holding a 40lb dumbbell in a "purse-carrying" position and that the tendons have a 50% efficiency loss in transmitting the motor torque to the fingers. From these calculations, it was found that the motors would need to output about 2Nm of torque in order to support the weight. Similarly, as with the power analysis, a Python script was written so that this calculation could be easily iterated upon in the future.

To accommodate the request of our project's sponsors, another motor torque analysis was performed. This other analysis differed from the first in that the goal was to investigate how much torque the motors would need to output in order to press a piano key. Given that it takes about 1Nm of force to press a piano key, this, along with the finger dimensions and tendon attachment points, served as the givens for a rudimentary statics problem. Similarly to the previous torque analysis, a 50% loss in efficiency through the tendons was assumed. Given the current dimensions of the fingers and a 20mm diameter pulley on the motor onto which the tendons attach and by which the motors will pull on the tendons, a minimum required torque of 0.144Nm was found. This is a pleasantly low number and shows that we can realistically attain the goals set forth by our sponsors

Tendons

David:

When making the decision of tendon material it was most important to analyze the finger that was under the most load which in our case is going to be the thumb. It was under the most load per length of the sections so based on the force we got for the thumb and the yield strength of various materials an area was calculated that could withstand the maximum force of each of the materials, Steel wire, Kevlar, and nylon could bear.

I first found the yield strength of the two most promising materials which were the kevlar cord and steel cable. I found those to be 2600 MPa for Kevlar and 1500 MPa for steel and then calculated the diameters based on a maximum force in the thumb of 111.25 N and the yield strengths of the materials in order to find which material could have the smallest diameter. The left represents the kevlar and right the steel wire.

$$A = \frac{111.25}{2600 \times 10^6} \approx 4.28 \times 10^{-5}\,\mathrm{m}^2 = 42.8\,\mathrm{mm}^2 \quad A = \frac{111.25}{1500 \times 10^6} \approx 7.42 \times 10^{-5}\,\mathrm{m}^2 = 74.2\,\mathrm{mm}^2$$

$$r = \sqrt{\frac{4.28 \times 10^{-5}}{\pi}} = \sqrt{1.36 \times 10^{-5}} \approx 3.69 \times 10^{-3}\,\mathrm{m} = 3.69\,\mathrm{mm} \quad r = \sqrt{\frac{7.42 \times 10^{-5}}{\pi}} = \sqrt{2.36 \times 10^{-5}} \approx 4.86 \times 10^{-3}\,\mathrm{m} = 4.86\,\mathrm{mm}$$

I then did a tendon pulley analysis based on a readjusted max force requirement that better suited our design of 40 N. This was done to make tendon size smaller as well as pulley size as we switched from tendon to pulley driven designs. We decided on a tendon diameter between 0.25mm and 0.5 mm but settled with 0.25 mm after doing a fatigue life analysis on both and receiving the best results from the 0.25mm diameter. These calculations were done on a more optimized tendon material in PBO which has a Youngs modulus of 270 GPa, S(ut) of 5800 MPa, and Se of 2900 MPa.

First finding cross sectional area, then tension and bending stressed, after that calculating pulley diameter.

$$D \geq 0.25 \cdot \frac{270,000 \times 2}{5,800} = 0.25 \times 93.1 = 23.3\,\text{mm}$$

Cross Sectional Area:

$$A = \frac{\pi}{4} \times (0.25)^2 = 0.049\,\text{mm}^2$$

Tension Stress:

$$\sigma_{\text{tension}} = \frac{40\,\text{N}}{0.049\,\text{mm}^2} = 816\,\text{MPa}$$

Bending Stress:

$$\sigma_{\text{bend}} = 270,000 \cdot \frac{0.25}{25} = 2,700\,\text{MPa}$$

I finally calculated the total stress and stress ratio and found the approximate cycles for our tendon material that is acceptable for our design.

Total Stress:

$$\sigma_{\text{total}} = 816 + 2,700 = 3,516\,\text{MPa}$$

Stress ratio:

$$\frac{3,516}{2,900} = 1.21 \rightarrow \sim 10^4 \text{ cycles}$$

API

Joseph:

Projectile motion equations prove useful twofold: The first application is to calculate the flight time of an object based on initial conditions, such as a launch angle of 30 degrees and a horizontal distance of 1.5 meters, the initial velocity is 4.122 meters per second. Using the initial velocity, the total flight time is calculated to be .42 seconds. This is assuming the same initial and final height. A time of .42 seconds equates to a frequency of .42 hertz, or .42 cycles per second, which is how fast the code will need to yield an adequate reaction speed.

Fingers

Joseph:

To best mimic natural position and range of motion of the hand, it was important to model the joint angles with certain grips, such as full actuation and gripping a ball with a diameter of 2.5 inches. After each joint and splay angle was tabulated, equations were built to link the movement of the middle joint to the movement of the tip joint by using a direct ratio. These equations can now be used in multiple ways; most notably, they can be implemented into a C++ script to allow the tip joint angle to be inferred based on the reading of an angle sensor and make the design easier by removing the need for motors at certain joints.

Additionally, the forward kinematics equations were utilized for a 3-link planar manipulator system and modified to represent the end effector's position over time by taking the time derivative of the equations. Once the equations were converted to matrix form and set equal to the x and y coordinate velocity vector, the matrix was solved to produce the Jacobian transformation matrix and the end-effector's x and y velocities in vector form. The calculations were automated with a MATLAB script, allowing the calculations to be iteratively solved to find the velocities of a fingertip over time. The calculation is as follows:

$$\begin{bmatrix} -L_1\sin(\theta_1) - L_2\sin(\theta_1+\theta_2) - L_3\sin(\theta_1+\theta_2+\theta_3) & -L_2\sin(\theta_1+\theta_2) - L_3\sin(\theta_1+\theta_2+\theta_3) & -L_3\sin(\theta_1+\theta_2+\theta_3) \\ L_1\cos(\theta_1) + L_2\cos(\theta_1+\theta_2) + L_3\cos(\theta_1+\theta_2+\theta_3) & L_2\cos(\theta_1+\theta_2) + L_3\cos(\theta_1+\theta_2+\theta_3) & L_3\cos(\theta_1+\theta_2+\theta_3) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

The time derivative of the forwards kinematics equation was taken to generate matrices. Using the previous dimensions of the fingers, a simple loop was created in C++.

$$L_1 = 8.7884\,\mathrm{cm}, L_2 = 4.9784\,\mathrm{cm}, L_3 = 4.2672\,\mathrm{cm}$$
$$\theta_1 = 56°, \theta_2 = -28°, \theta_3 = -10°$$
$$\dot{\theta}_1 = 30°/\mathrm{s}, \dot{\theta}_2 = 10°/\mathrm{s}, \dot{\theta}_3 = -10°/\mathrm{s}$$

The code then yields:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -6.1369\,\mathrm{cm/s} \\ 7.7667\,\mathrm{cm/s} \end{bmatrix}$$

The resulting matrix represents the fingertip's movement in a two-dimensional plane over time. Values can be substituted for other finger dimensions and positions.

Justin:

Using inverse kinematics assuming a 2 joint connection, the length of each segment and the end position of the tip the angle at each joint segment could be found which could be useful when programming the finger. It will need to be able to find the angles needed to end at a specific point in a 2D plane.

Joint 1	Joint 2
$X_1=L_1\cos(\Theta_1)$ $Y_1=L_1\sin(\Theta_1)$	$X_2=X_1+ L_2\cos(\Theta_1+ \Theta_2)$ $Y_2=Y_1+L_2\sin(\Theta_1 + \Theta_2)$

Using forward kinematics to analyze a 3 joint connection with a splaying motion, we assume the length of each segment to be proportional to 1.5x human fingers (Table 3) and the angle of each segment to be whatever we want it to be as long as it is within human range of motion limits. We will use the forward kinematics equations to program the finger because we could tell it what angles to bend at, and we could then know the finger's end position in a 3D space which replicates what it will need to do.

Joint 1	Joint 2	Joint 3
$X_1=L_1\cos(\Theta_1)$ $Y_1=L_1\sin(\Theta_1)$ $Z_1= L_1\sin(\boldsymbol{\Phi}_1)$	$X_2=X_1 + L_2 \cos(\Theta_1 + \Theta_2)$ $Y_2 =Y_1 +L_2 \sin(\Theta_1 + \Theta_2)$ $Z_2 =Z_1 + L_2 \sin(\boldsymbol{\Phi}_1 + \boldsymbol{\Phi}_2)$	$X_3= X_2+L_3\cos(\Theta_1+ \Theta_2+ \Theta_3)$ $Y_3= Y_2+L_3\sin(\Theta_1 + \Theta_2+ \Theta_3)$ $Z_3=Z_2 + L_3\sin(\boldsymbol{\Phi}_1 +\boldsymbol{\Phi}_2+\boldsymbol{\Phi}_3)$

David:

Finding a material for gripping with a proper coefficient of friction that is high enough to keep forces on fingers, joints, and tendons was essential for our design process. If we were to use rubber with a coefficient of friction on 0.8 that alone would lower our maximum allowable weight down significantly, but it would require the same amount of force on tendons and joints, so out calculated tendon sizes would be compromised much earlier than anticipated. We then factored in a safety factor of 1.5 to make sure that we would have plenty of room for error on these calculations and came to a final grip force of around half of what was initially calculated for.

Based on our max allowable load on fingers and the coefficient of friction we got a readjusted maximum allowable load shown below.

$$F_f = 0.8 \times 106\,\mathrm{N} = 84.8\,\mathrm{N}$$

I then took the recalculated grip force and applied the 1.5 safety factor to it which gave a final resultant force which I then made into a maximum allowable weight for the fingers.

$$F_{\text{grip, design}} = \frac{84.8\,\mathrm{N}}{1.5} = 56.5\,\mathrm{N}$$

$$m_{\text{design}} = \frac{56.5\,\mathrm{N}}{9.81\,\mathrm{m/s^2}} = 5.76\,\mathrm{kg}$$

Tyler:

For this calculation, I used the Denavit-Hartenberg parameters which is used to assign coordinate frames and parameters to each link and joint of a finger. Using some given information like lengths of the segments of the fingers and some sample angles of the joints, we can find the location in x and y axis of where the fingertip is in relationship to the base of the finger. The equations used below are what i used for this modeling. We can use any measurements or givens to find the positioning when implementing this into our code.

Using these given lengths and angles below as an example I was able to solve and example problem:

$L_1 = 5\ cm$

$L_2 = 3\ cm$

$\theta_1 = 45° \ or \ \frac{\pi}{4}$

$\theta_2 = 30° \ or \ \frac{\pi}{6}$

Link	Link Length	Link Twist	Offset	Joint Angle
1	L_1	0	0	θ_1
2	L_2	0	0	θ_2

Then using the following calculations by the Denavit-Hartenberg parameters, i was able to get to matrices from the base to link 1 and link 1 to link 2

Base to Link 1:

$$T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & L_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & L_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Link 1 to Link 2:

$$T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally, using matrix multiplication, I was able to get a result:

$$T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & L_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & L_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$$
$$y = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2)$$

$$\begin{aligned} y &= 5\sin\left(\frac{\pi}{4}\right) + 3\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) \\ &= 5\left(\frac{\sqrt{2}}{2}\right) + 3\left(\frac{1}{2}\right) \\ &\approx 3.54 + 1.50 \\ &\approx 5.04\,\text{cm} \end{aligned}$$

$$\begin{aligned} x &= 5\cos\left(\frac{\pi}{4}\right) + 3\cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) \\ &= 5\left(\frac{\sqrt{2}}{2}\right) + 3\left(\frac{\sqrt{3}}{2}\right) \\ &\approx 3.54 + 2.60 \\ &\approx 6.14\,\text{cm} \end{aligned}$$

I was able to find the location of the fingertip in relation to the angles and lengths of each joint. This gave me a position of (6.14 cm, 5.04 cm).

Overall Measurements

Tyler:

For this calculation I got the measurements of every part of the hand. We need this to be able to do any modeling and any calculations for the hand. I measured my hand and wrote down all of the measurements in Table 2 below. I also included an upper limit of each measurement.

Table 2: Overall measurements for Hand

	Length(inches)	width (inches)	Other (inches)	Length upper Limit	width upper length	other upper limit
overall length	7.6			11.4		
overall breadth		3.5			5.25	
average circumference			8.6			12.9
Index Finger	4.125			6.1875		
top segment	1.125	0.625		1.6875	0.9375	
middle segment	1.125	0.75		1.6875	1.125	
base segment	1.875	0.875		2.8125	1.3125	
Middle finger	4.75			7.125		
top segment	1.125	0.625		1.6875	0.9375	
middle segment	1.3125	0.75		1.96875	1.125	
base segment	2.3125	0.875		3.46875	1.3125	
Ring finger	4.5			6.75		
top segment	1.125	0.625		1.6875	0.9375	
middle segment	1.0625	0.75		1.59375	1.125	
base segment	2.1875	0.875		3.28125	1.3125	
Pinky Finger	3.5			5.25		
top segment	0.9375	0.5		1.40625	0.75	
middle segment	0.875	0.625		1.3125	0.9375	
base segment	1.6875	0.75		2.53125	1.125	
Thumb	2.875			4.3125		
Top segment	1.375	1		2.0625	1.5	
base segment	1.5	0.875		2.25	1.3125	
Palm	4.75	3.5		7.125	5.25	

Motor Speed

Markus:

To determine the required motor speed for the robotic hand's tendon-driven actuation, we performed calculations based on reaction time and finger displacement. Motor speed is critical to achieving the rapid movement needed for tasks like playing the piano and catching a ball. The following calculations establish the appropriate speed based on tendon displacement and time constraints.

Given:

- Tendon displacement range: 45-75 mm
- Total reaction time: 300 ms (assuming 25 ms for signal processing)
- Spool radius options: 5 mm and 10 mm

Using the equation:

$$\omega = \frac{d}{rt}$$

where:

- ω = angular velocity (rad/s)
- d = displacement (m)
- r = spool radius (m)
- t = time (s)

For a 5mm spool:

$$\omega = \frac{.075}{.005 \cdot .275} \approx 278$$

[Equation 1]

For a 10mm spool:

$$\omega = \frac{.075}{.01 \cdot .275} \approx 139$$

[Equation 2]

A larger spool reduces the required motor RPM but increases torque demand.

Shear Stress in Joints

Markus:

To ensure the structural integrity of the robotic hand, we calculated the shear stress experienced at each joint due to tendon force transmission. The goal is to verify that the selected materials can withstand the forces exerted during actuation without failure. These calculations help in selecting the appropriate materials for longevity and reliability.

$$V = \frac{T}{r} + F$$

And

$$\tau = \frac{F}{A}$$

where:

- $F =$ Applied force at fingertip (N)
- $T =$ Applied torque at joint (Nm)
- $r =$ radius of torque(m)
- $\tau =$ shear stress at joint (Pa)
- $A =$ cross-sectional area of pin joint (m^2)

Based on the CAD model of our pin, the smallest cross section is 10.16 mm^2. This calculation is for the first joint in the thumb as this experiences the most shear force. The torque is given based on the position of the fingers within the hand and the weight the hand is holding. The calculations are done assuming a 40lb grip force.

$$V = \frac{2.225}{.01} + 44.475 = 266.975N$$

$$\tau = \frac{266.975}{10.16} = 26.28\, MPa$$

Carbon fiber can have shear strength up to 27MPa, further analysis needs to be done to ensure carbon fiber pins will be sufficiently strong.

Table 3: Complete List of Shear in Each Finger

Joint	# Torque (T) (Nm)	# Shear Force V (N)	# Shear Stress τ (MPa)
Thumb 1	2.225	266.975	26.28
Thumb 2	1.335	177.975	17.52
Thumb 3	0.665	110.975	10.92
Index 1	1.6	213.37	21.00
Index 2	1.065	159.87	15.74
Index 3	0.535	106.87	10.52
Middle 1	0.8	106.685	10.50
Middle 2	0.535	80.185	7.89
Middle 3	0.265	53.185	5.23
Ring 1	0.8	106.685	10.50
Ring 2	0.535	80.185	7.89
Ring 3	0.265	53.185	5.23
Pinky 1	0.535	71.29	7.02
Pinky 2	0.355	53.29	5.25
Pinky 3	0.18	35.79	3.52

Torque in Joints

David:

To ensure the proper material choices for the robotic hand and finger components it was necessary to calculate the torque that would be applied to each one of the joints on each of the fingers. We based this on an average grip force distribution that focused more of the force on the thumb, index, and middle fingers so they were the three most important torques to calculate. We based our gripping force on the average grip force of a human which came out to be around 80lb or 36kg.

Thumb force $F_{\text{thumb}} = 0.25 \times 355.8\,\text{N} = 88.95\,\text{N}$

Index force $F_{\text{index}} = 0.30 \times 355.8\,\text{N} = 106.74\,\text{N}$

Middle force $F_{\text{middle}} = 0.15 \times 355.8\,\text{N} = 53.37\,\text{N}$

Ring force $F_{\text{ring}} = 0.15 \times 355.8\,\text{N} = 53.37\,\text{N}$

Pinky force $F_{\text{pinky}} = 0.10 \times 355.8\,\text{N} = 35.58\,\text{N}$

I then found torques in each of the joints by assuming a 1 cm distance from joint to joint for the fingers but the thumb, which is more accurate for our design than it is to a human finger.

A. Thumb:

$\tau_{\text{thumb, joint 1}} = 88.95 \cdot 0.05 = 4.45\,\text{N}\cdot\text{m}, \quad \tau_{\text{thumb, joint 2}} = 88.95 \cdot 0.03 = 2.67\,\text{N}\cdot\text{m}, \quad \tau_{\text{thumb, joint 3}} = 88.95 \cdot 0.015 = 1.33\,\text{N}\cdot\text{m}$

B. Index:

$\tau_{\text{index, joint 1}} = 106.74 \cdot 0.03 = 3.20\,\text{N}\cdot\text{m}, \quad \tau_{\text{index, joint 2}} = 106.74 \cdot 0.02 = 2.13\,\text{N}\cdot\text{m}, \quad \tau_{\text{index, joint 3}} = 106.74 \cdot 0.01 = 1.07\,\text{N}\cdot\text{m}$

C. Middle:

$\tau_{\text{middle, joint 1}} = 53.37 \cdot 0.03 = 1.60\,\text{N}\cdot\text{m}, \quad \tau_{\text{middle, joint 2}} = 53.37 \cdot 0.02 = 1.07\,\text{N}\cdot\text{m}, \quad \tau_{\text{middle, joint 3}} = 53.37 \cdot 0.01 = 0.53\,\text{N}\cdot\text{m}$

D. Ring:

$\tau_{\text{ring, joint 1}} = 53.37 \cdot 0.03 = 1.60\,\text{N}\cdot\text{m}, \quad \tau_{\text{ring, joint 2}} = 53.37 \cdot 0.02 = 1.07\,\text{N}\cdot\text{m}, \quad \tau_{\text{ring, joint 3}} = 53.37 \cdot 0.01 = 0.53\,\text{N}\cdot\text{m}$

E. Pinky:

$\tau_{\text{pinky, joint 1}} = 35.58 \cdot 0.03 = 1.07\,\text{N}\cdot\text{m}, \quad \tau_{\text{pinky, joint 2}} = 35.58 \cdot 0.02 = 0.71\,\text{N}\cdot\text{m}, \quad \tau_{\text{pinky, joint 3}} = 35.58 \cdot 0.01 = 0.36\,\text{N}\cdot\text{m}$

Wrist Actuation

Tyler:

When looking into how to actuate our wrist, we landed on the idea of using gears to actuate it. Using gear ratio equations to find the right sized gears, we found that the correct ratio should be 2:1. This will allow for higher torque, more precision, but slower movement. As this is the wrist I felt that the speed of the wrist is not overly important. The approximate number of teeth of these gears should be 40 and 20 teeth. Also using this we found an output RPM of 139. Below is what the equation looks like;

$$GR = \frac{\text{Output Speed}(\omega_o)}{\text{Input Speed}(\omega_i)}$$

$$GR = \frac{Z_o}{Z_i}$$

$$\omega_o = \frac{278}{2} = 139\ \text{RPM}$$

Equation Summary

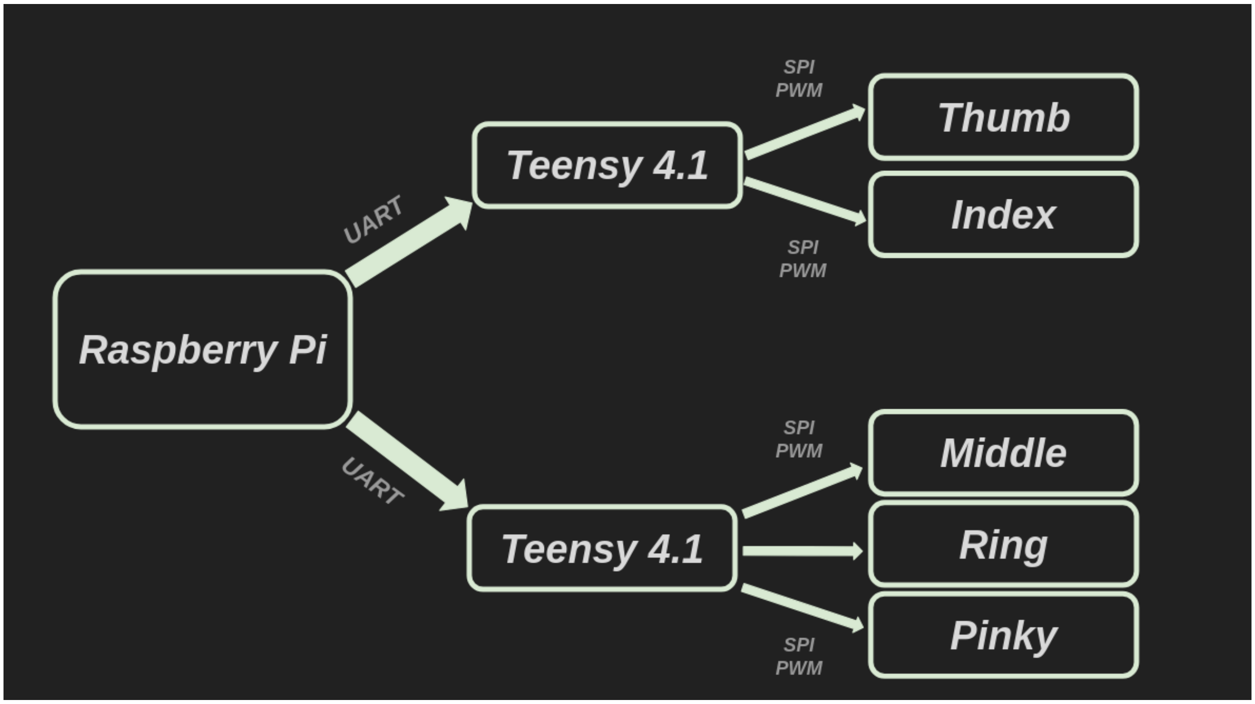
All calculations are here and are summarized to make sense:

Table 4: Summary of Calculations

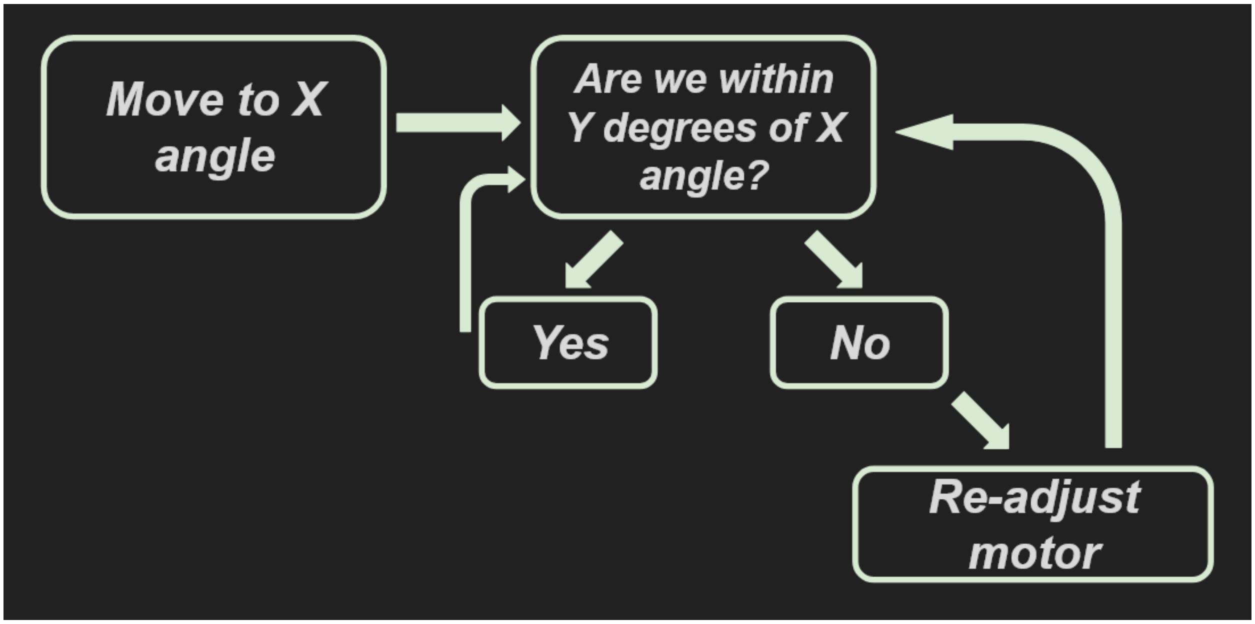
Calculation	*Equation(s)*	*Application*	*Requirement Met*	*Validation*

Projectile motion	$x_f=x_0+v_{0x}t$ $x_f=(v^2 sin2theta)/g$	*Catching a ball*	*Dexterity and reaction speed*	*Dynamics* *Assumptions*
Finger tip joint inference	$theta_{Tip}=.667theta_{Mid}$ $theta_{Tip}=.556theta_{Mid}$ $theta_{Tip}=.333theta_{Mid}$	*Coding, ease of design, mechanical linkages*	*Biomimetic and natural motion*	*Speculation* *Grip Angles*
Motor Speed		*For Motor Selection*	*Hand actuation speed*	*Speculation* *Reaction time*
Shear Stress		*For material selection for joints*	*Number of actuations*	*Speculation* *Average Material*
Fingertip location (x,y,z)	$x_3 = x_2 + L_3 \cos(\theta_1 + \theta_2 + \theta_3)\cos(\phi)$ $y_3 = y_2 + L_3 \sin(\theta_1 + \theta_2 + \theta_3)\cos(\phi)$ $z_3 = z_2 + L_3 \sin(\phi)$	*Finding location of fingertip in terms of the base joint*	*Control of the fingers*	*Implementing code* *Real finger lengths*
Hand Measurements	*N/A*	*Have exact measurements of joints and segments*	*Average hand size and upper limit*	*Speculation* *Average Measurements*
Power	*P = V*I*	*Power consumption*	*Reasonable power consumption*	*Equations used agree with what was learned in PHY 262, EE188* *Compare results to power consumption of real-world electrical devices*
Motor Torque	*F = ma* *T = Fr*	*Inform motor selection*	*Establish minimum required motor torque*	*Equations and their application agree with the basic principles of static analysis* *Required motor torque agrees with reason*
Wrist Actuation	$GR = \frac{\text{Output Speed}(\omega_o)}{\text{Input Speed}(\omega_i)}$ $GR = \frac{Z_o}{Z_i}$ $\omega_o = \frac{278}{2} = 139\text{ RPM}$	*Moving the wrist*	*Actuation process for the wrist*	*Using Machine design equations and knowledge from machine design.*
End-Effector Velocity	[illegible]	*Determining the velocity vector of a fingertip*	*Speed and dexterity*	*Previous kinematic equations and literature*

Flow Charts and other Diagrams:



The flow diagram above provides an outline of the general control scheme of the hand. At the highest level, the Raspberry Pi will send out commands to the two Teensy 4.1s. These commands will be abstract commands such as ”flex all fingers with x amount of force” or ”perform pencil grip.” The Teensies then take these commands and translate them into commands that the motors understand and send them via SPI to the BLDC motors in the forearm for finger flexion/extension movements and via PWM to the servos in the hand which handle splaying motion. While the fine details of the control scheme are still being worked out, this flow diagram shows the high-level control scheme.



This flow diagram depicts the closed-loop control scheme for the fingers. The joints of the fingers will have potentiometers which report their angular positions back to the Teensies. The Teensies will compare the potentiometer values to the desired values and command the motors to adjust accordingly.

The desired position prescribed by the Teensies will have a margin for error to prevent excessive "jittering" of the fingers caused by the motors continually trying to achieve the perfect desired position. This margin for error will be determined experimentally through testing of the system for a balance between accuracy and smooth control.

These flow diagrams provide an outline of the general control scheme and serve as a starting-off point for software development. The finer details of control will be defined as the software of the hand develops.

Moving Forward:

The next steps to take to ensure a successful project include finalizing a finger design that incorporates a mechanical link between second and third joints. A mechanical linkage from the fingertip to the middle finger segment would greatly simplify control and eliminate extra motors that would otherwise be required to individually actuate each joint. Two possible avenues to take to accomplish such a linkage are using a torsional spring or implementing a true mechanical link inspired by research and literature. If the spring option is decided to implement, calculations for the spring force acting on the fingertip and ensure the motors can overcome the spring force so the finger bends when actuated while preserving the motor. Furthermore, the control scheme code will be finalized during the majority of the remaining semester. The code will allow a method of communication to fully actuate the hand with a level of control that meets the applicable engineering and customer requirements.